OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS   **2608/1**

Mechanics 2

Friday   **14 JANUARY 2005**   Morning   1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Where a numerical value for the acceleration due to gravity is needed, use g = 9.8 m s^{-2} unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 A uniform beam AB of length 3 m and weight 80 N is freely hinged at A.

Initially, the beam is held horizontally in equilibrium by a small, smooth peg at C where the distance AC is 2.5 m, as shown in Fig. 1.1.
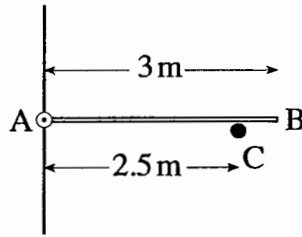

Fig. 1.1

(i) Calculate the force on the beam from the peg at C. [3]

The peg is now moved so that the beam is in equilibrium with AB at 60° to the vertical, as shown in Fig. 1.2. AC is still 2.5 m.
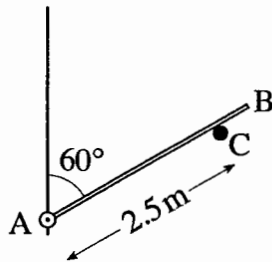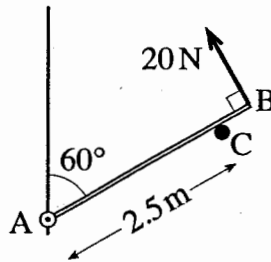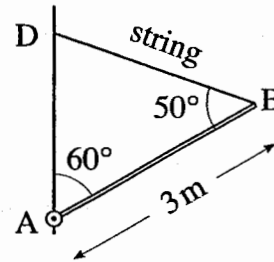

Fig. 1.2   **Fig. 1.3**   **Fig. 1.4**

(ii) Calculate the new force on the beam due to the peg. [4]

A light string is now attached to the beam at B. The string is perpendicular to the beam. The beam is in equilibrium with a tension of 20 N in the string, as shown in Fig. 1.3.

(iii) Calculate the new force on the beam due to the peg. [2]

The peg is now removed and the string attached to a point D vertically above A so that angle ABD is 50°, as shown in Fig. 1.4.

(iv) Calculate the new tension in the string. Calculate also the vertical component of the force acting on the hinge at A. [5]

[Total 14]

2 A small block of mass 25 kg is on a rough slope inclined at $\alpha°$ to the horizontal. The block is held in equilibrium by a force of magnitude P N applied parallel to the slope and up the slope, as shown in Fig. 2.
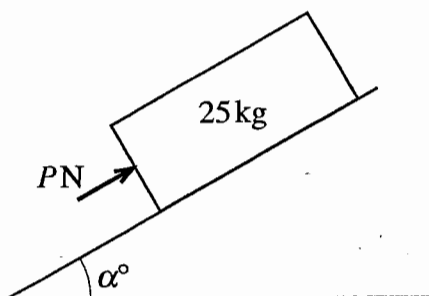

Fig. 2

When $P = 259$, the block is about to slip **up** the slope. When $P = 35$, the block is about to slip **down** the slope. In each case, the magnitude of the frictional force acting on the block is F N.

(i) Draw separate force diagrams for these two cases, showing all the forces and making clear the direction in which the frictional force acts. [2]

(ii) Calculate the value of α and show that $F = 112$. [5]

(iii) Calculate the coefficient of friction between the block and the slope. [3]

The force of magnitude P N is removed and the block slides from rest down the slope. The slope is not uniformly smooth but the frictional force averages 112 N over the distance slid.

The speed of the block is to be calculated after it has slid 3 m.

(iv) (A) Explain briefly why a method using Newton's second law with the constant acceleration formulae is not appropriate.

(B) Use an energy method to calculate the speed. [6]

[Total 16]

[Turn over

3 A lamina ACDEFI is made from uniform material. ACGI and DEFG are both rectangular. The dimensions in centimetres are shown in Fig. 3.1.
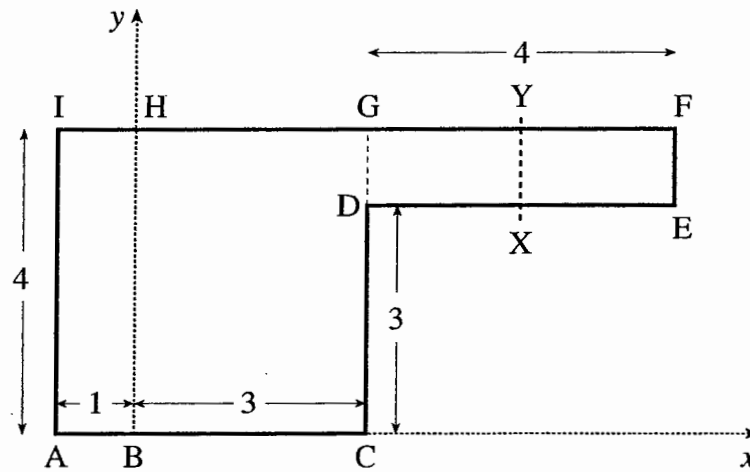

Fig. 3.1

(i) Calculate the coordinates of the centre of mass of the lamina ACDEFI, with respect to the axes shown in Fig. 3.1. [5]

ABHI is now folded along BH so that it is perpendicular to the plane BCGH. XEFY is folded along XY so that it is also perpendicular to the plane BCGH but on the other side of it. This situation is shown in Fig. 3.2. XY is parallel to EF and the distance EX is such that the centre of mass of the folded lamina remains in the plane BCGH.

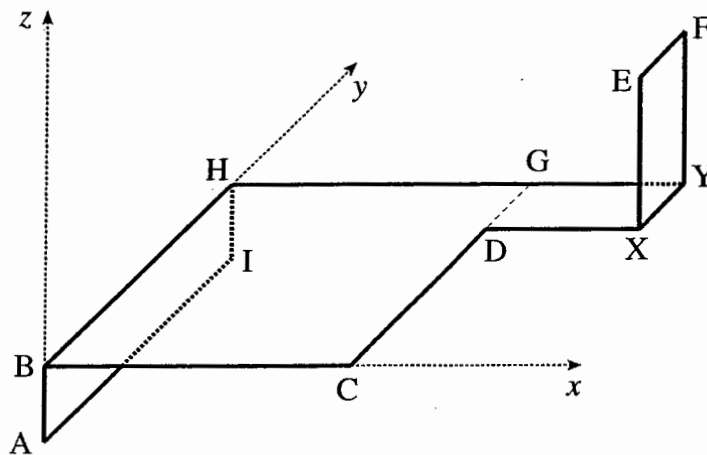**Fig. 3.2**

(ii) Verify that the distance EX is 2 cm. [4]

(iii) Calculate the x- and y-coordinates of the centre of mass of the folded lamina with respect to the axes shown in Fig. 3.2. [4]

(iv) The folded lamina is freely suspended from H and hangs in equilibrium with HBCDXYG in a vertical plane. Calculate the angle between HB and the vertical. [3]

[Total 16]

4 Two circular discs of equal radius slide on a smooth, horizontal surface. Disc A has mass 2 kg and disc B has mass m kg. All impacts are direct and motion is along the line of centres, which is perpendicular to a wall. The situation is shown in Fig. 4.
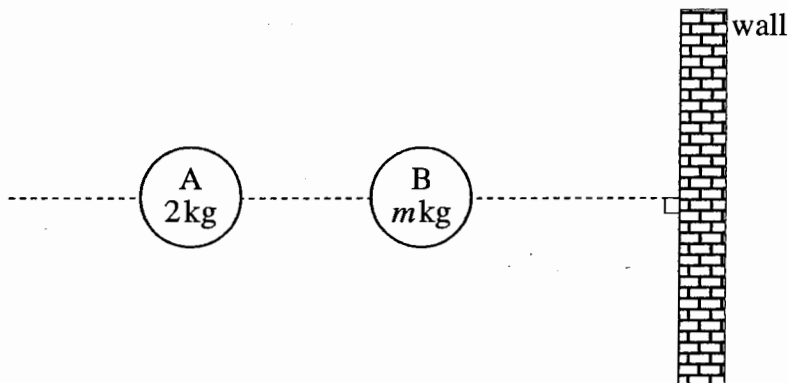

Fig. 4

A force of 4 N acts on A for 5 seconds along the line of centres in the direction AB. Disc A is initially at rest and does not reach disc B in the 5 seconds.

(i) Show that A achieves a speed of $10\,\text{m s}^{-1}$ in the direction AB. [1]

Suppose that $m = 5$, B is initially at rest and the coefficient of restitution in the impact between A and B is 0.54.

(ii) Show that, after the impact, A has a speed of $1\,\text{m s}^{-1}$ in the direction BA and B has a speed of $4.4\,\text{m s}^{-1}$ in the direction AB. [5]

Disc B rebounds from the wall with its speed halved.

(iii) Calculate the impulse of the wall on B. [2]

Consider now the more general case of the collision between A and B where B is initially at rest. In this general case, B has mass m kg and the coefficient of restitution in the impact between A and B is e. Disc A travels at $10\,\text{m s}^{-1}$ in the direction AB and collides with B. After the collision, A has speed $1\,\text{m s}^{-1}$ in the direction BA.

(iv) **(A)** Show that $e = \dfrac{22 + m}{10m}$.

(B) The speed of B is halved in its impact with the wall. Disc B collides again with A after the impact with the wall. Show that $m < 11$ and find the range of possible values of e. [6]
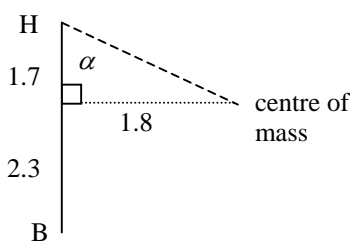
[Total 14]

Mark Scheme

Solutions and mark scheme

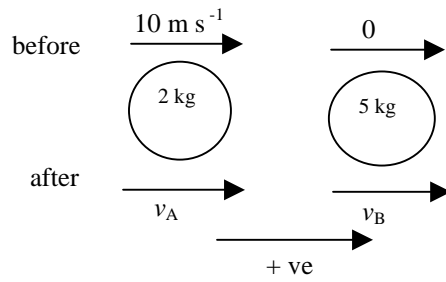| Q 1		mark		sub
(i)	Moments c.w. about A $80 \times 1.5 = 2.5 R_C$ $R_C = 48$ so 48 N	M1 A1 A1	Attempt at use of moments. Correct	 3
(ii)	Moments c.w. about A $80 \times 1.5 \times \sin 60 = 2.5 R_C$ $R_C = 24\sqrt{3}$ $= 41.5692\ldots$ so 41.6 N (3 s. f.)	M1 B1 B1 A1	Attempt at moments equation and resolving LHS correct RHS correct	 4
(iii)	Moments c.w. about A $80 \times 1.5 \times \sin 60 = 2.5 R_C + 3 \times 20$ $R_C = 24\left(\sqrt{3} - 1\right) = 17.569...$ so 17.6 N (3 s. f.)	M1 A1	Equn in (ii) with 3×20 term attempted. cao	 2
(iv)	$80 \times 1.5 \times \sin 60 = T \times 3 \sin 50$ $T = 45.2206\ldots$ so 45.2 N (3 s. f.) $\uparrow\ Y + T\cos 70 = 80$ $Y = 64.533..$ so 64.5 N (3 s. f.) upwards	M1 A1 A1 M1 A1	Moments equation with all terms present and with attempt at resolution of T. Correct resolution of T Resolve vertically (or take moments again) with all forces present. cao. Accept no direction	 5
			total	14

Solutions and mark scheme

Q 2		mark		sub
(i)		B1	All forces present with arrows and labels. Angle not required	
		B1	All forces present with arrows and labels. Angle not required.	2
(ii)	up $\quad 259 = 25g\sin\alpha + F$	B1		
	down $\quad F + 35 = 25g\sin\alpha$	B1		
	Eliminate F $259 = 50g\sin\alpha - 35$	M1	Eliminate one variable	
	so $\sin\alpha = 0.6$ and $\alpha = 36.869\ldots$ so 36.9 (3 s. f.)	A1		
	Back substitute $F = 25 \times 9.8 \times 0.6 - 35 = 112$	E1		5
(iii)	$F = \mu R$	M1	Must be substituted	
	$R = 25g\cos\alpha$	B1	FT **their** 0.8 from α	
	$\mu = \dfrac{112}{25 \times 9.8 \times 0.8} = \dfrac{112}{196} = \dfrac{4}{7} \ (= 0.571428\ldots)$	A1	cao	3
(iv) (A)	The forces are not constant so acceleration is not constant. (Use of the method would have to be justified.)	E1		
(B)	$0.5 \times 25 \times v^2 = 25g \times 3 \times 0.6 - 3 \times 112$	M1 B1 B1 B1	W-E equation. All terms present KE term GPE term. FT **their** 0.6 from α WD against friction	
	$v^2 = 8.4$ so $v = 2.8982\ldots$ so 2.90 m s^{-1} (3s.f.)	A1	cao	6
			total	16

Solutions and mark scheme

Q 3		mark		sub
(i)	$$(16+4)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 16\begin{pmatrix} 1 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 5 \\ 3.5 \end{pmatrix}$$	M1	Method for c.m.	
		B1	Correct masses	
		B1	Correct c.m. of one cpt or both x or both y cpts correct	
	$\bar{x} = 1.8$	A1		
	$\bar{y} = 2.3$	A1		
			[Award up to 4/5 if other axes used]	5
(ii)	$\bar{z} = 0$	M1	Attempt at z cpt of c.m. of flaps to get $\bar{z} = 0$	
	ABHI XEFY			
	$4 \times \dfrac{1}{2} = EX \times \dfrac{EX}{2}$	B1	LHS	
		B1	RHS	
	sub EX = 2 giving $2 = 2 \times 2 / 2$	E1	Justified	
			[Award 2/4 for $4 \times \dfrac{1}{2} = 2 \times \dfrac{EX}{2} \Rightarrow EX = 2$]	4
(iii)		M1	FT use of wrong axes Appropriate method to deal with flaps	
	$20\bar{x} = 4 \times 0 + 12 \times 1.5 + 2 \times 4 + 2 \times 5 = 36$	B1	Any non-zero component of $\bar{x}$ correct	
	$\bar{x} = 1.8$ (the same)	A1		
			[Award M1 A0 A1 for 'the same' stated WW]	
	$\bar{y}$ remains 2.3	B1		4
(iv)		B1	Identify required angle and relate to $(\bar{x}, \bar{y})$. May be implied by correct expression for α.	
	$\tan \alpha = \dfrac{\bar{x}}{4 - \bar{y}}$	M1	Use of arctan expression. Accept only correct expression or $\tan \alpha = \dfrac{\bar{x}}{\bar{y}}$ or $\tan \alpha = \dfrac{\bar{y}}{\bar{x}}$, using **their** values.	
	$\alpha = 46.636\ldots$ so $46.6°$ (3 s. f.)	A1	cao. Any reasonable accuracy	3
			total	16

Solutions and mark scheme

Q 4		mark		sub
(i)	$Ft = 4 \times 5 = 20$ N s $20 = 2v$ so $v = 10$	E1		1
(ii)	 PCLM $\quad 20 = 2v_A + 5\,v_B$ NEL $\quad \dfrac{v_B - v_A}{0 - 10} = -0.54$ Either solve or substitute values in **both** equations to give $v_A = -1$ and $v_B = 4.4$	M1 A1 M1 A1 E1	Use of PCLM Any form Use of NEL Any form Directions must be clearly established (accept diagram).	5
(iii)	$\leftarrow \quad 5(2.2 - (-4.4))$ $\quad\quad = 33$ N s	M1 A1	$I = m(v - u)$ attempted Accept ± 33 N s	2
(iv) (A)	NEL $\quad \dfrac{v_B + 1}{-10} = -e$ so $v_B = 10e - 1$ PCLM $\quad 20 = -2 + mv_B$ so $v_B = \dfrac{22}{m}$ Eliminating v_B gives $e = \dfrac{22 + m}{10m}$	M1 M1 E1	Use of NEL. Allow only sign errors Use of PCLM. Allow only sign errors Clearly shown	
(B)	Need $v_B > 2$ so $m < 11$ Hence $e > \dfrac{\frac{22}{11} + 1}{10} = 0.3$ so $0.3 < e \leq 1$	E1 M1 E1	Argue that $m < 11$ Reorganise inequality to show result (accept use of =) Clearly shown (the inequality must be justified) $e \leq 1$ is not required	6
			total	14

Examiner's Report